

Lec (11) 23

Lattice matched Hetero structure:

Strained layer Epitaxy:

atoms of grown layer adjust themselves to be matched with atoms of substrate

Condition: thickness of grown layer $< d_c$

$$d_c = \frac{a_s}{2\epsilon}$$

$$\epsilon = \frac{a_L - a_s}{a_L}$$

↓

mismatch material

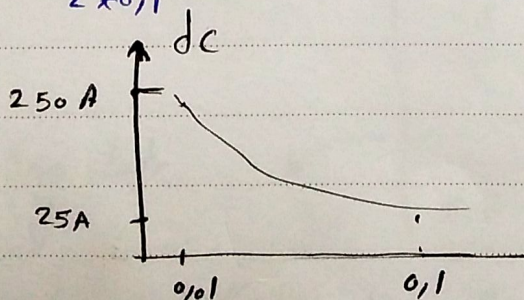
Ex:

$$\epsilon = 0,01 \quad a_s = 5 \text{ \AA}$$

$$d_c = \frac{5}{2 \times 0,01} = 250 \text{ \AA}$$

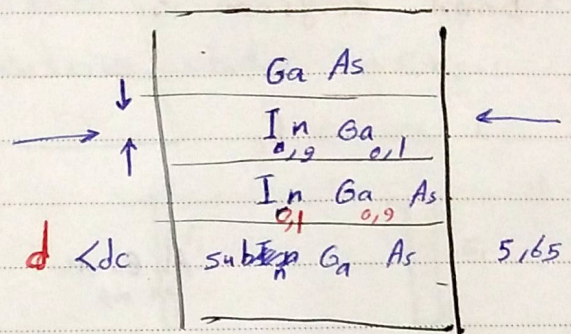
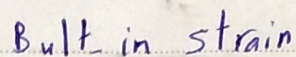
$$\text{Ex} \quad \epsilon = 0,1 \quad a_s = 5 \text{ \AA} \quad a_L = \frac{5}{0,9} \text{ \AA}$$

$$d_c = \frac{5}{2 \times 0,1} = 25 \text{ \AA}$$



EX: $\text{In}_{\frac{x}{1-x}} \text{Ga}_{\frac{1-x}{1-x}} \text{As} / \text{GaAs}$

$X=0$ GaAs $a=5,65 \text{ \AA}$

$$X = 1 \quad I_{ns} \quad a = 5,87 \text{ Å}^0$$


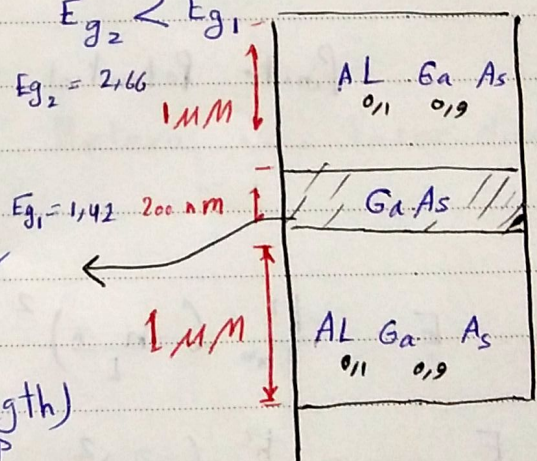
Double Hetero structure:

two semiconductor material grown into a sandwich

type 1 outer layer $\rightarrow$ material has E_g

The sandwich layer has $E_{g2} < E_{g1}$

Ex: $AL_xGa_{1-x}As$ / $GaAs$



Active layer

electron wavelength

Thickness $d < \lambda$ (deBroglie wave length)

$$\lambda = \frac{2\pi}{k}$$

$$K = \frac{P}{h}$$

$$E_{K.E} = \frac{1}{2} m v^2 \cdot \frac{m}{m}$$

$$E_{KE} = \frac{1}{2} \frac{(mv)^2}{m} = \frac{P^2}{2m}$$

$$\Rightarrow p = \sqrt{2m^*E} \rightarrow = k \cdot \lambda$$

$$E_{\text{thermal energy}} = 0,025 \text{ eV}$$

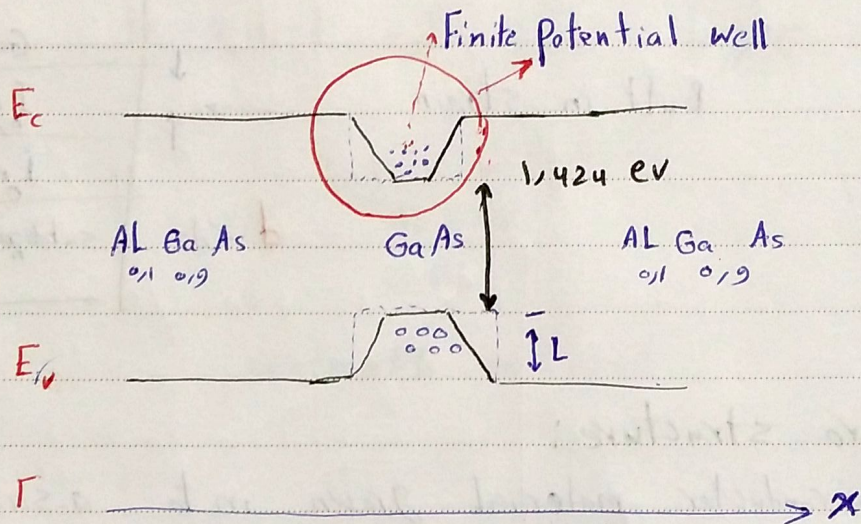
$$m_c^* = 0,067 m_0 = 0,067 \times 9,1 \times 10^{-31}$$

$$\Lambda = 300 \text{ \AA}$$

* $\Rightarrow$ to convert to quantum well structure from bulk material
 $\Rightarrow d_L < (\lambda_e) \Lambda$

Band diagram :

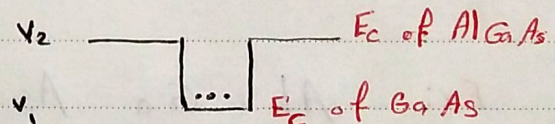
electron in a box



$$E_g|_{x=0,1} = 1,424 + 1,207x = 1,55 \text{ eV}$$

$$x < 0,45$$

finite Potential Well (finite Quantum well)



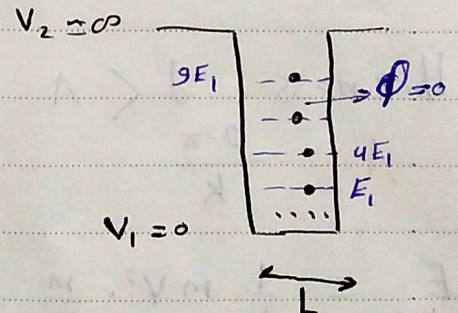
in finite Potential

$$E = \frac{\hbar^2}{2m^*} \left(\frac{n\pi}{L} \right)^2$$

$$n = 1, 2, 3, \dots$$

$$E_{n=1} = \frac{\hbar^2}{2m^*} \left(\frac{\pi}{L} \right)^2$$

$$E_{n=2} = 4 E_1$$



$$\begin{aligned} L \uparrow &\Rightarrow E \downarrow \\ L \downarrow &\Rightarrow E \uparrow \end{aligned}$$

finite Potential well :

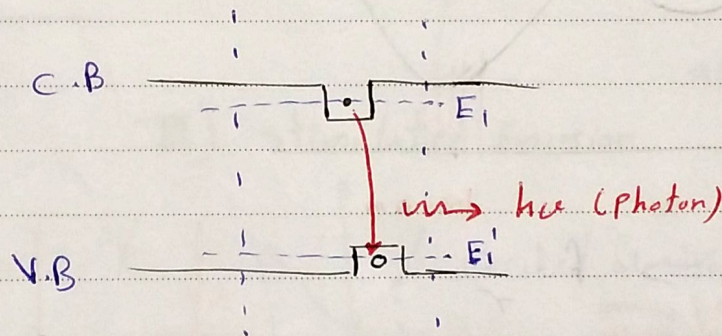
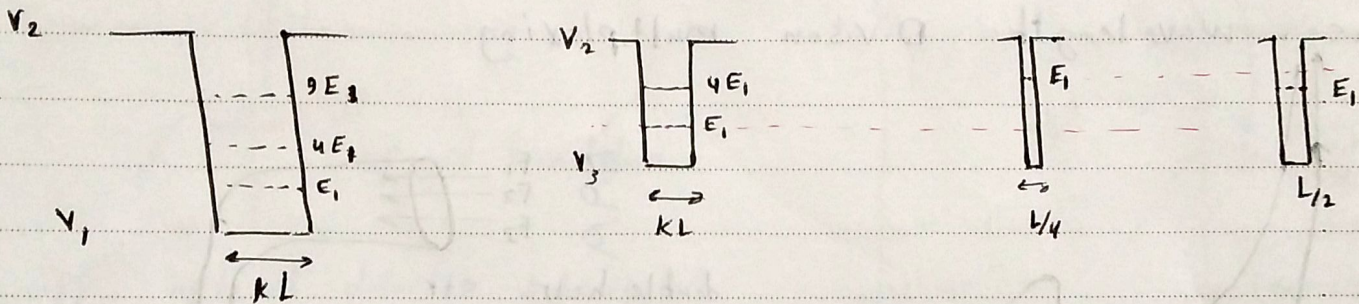
$$E_{c2} > E_n > E_{c1}$$

As L (thickness of active material)

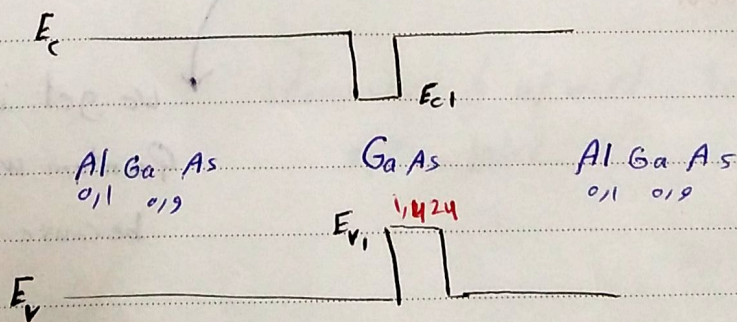
Q4
sheet 8

the thickness (L)

decreases $\Rightarrow E_1$ the first state in the well shift up
& distance between successive state increases



The application of Double Hetero is laser diode



$$E_{c1} = 0.67 E_g'$$

$$(E_{g' AlGaAs} - E_{g GaAs})$$

$$E_{v1} = 0.33 E_g'$$

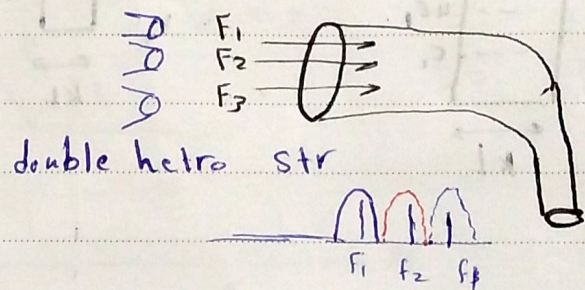
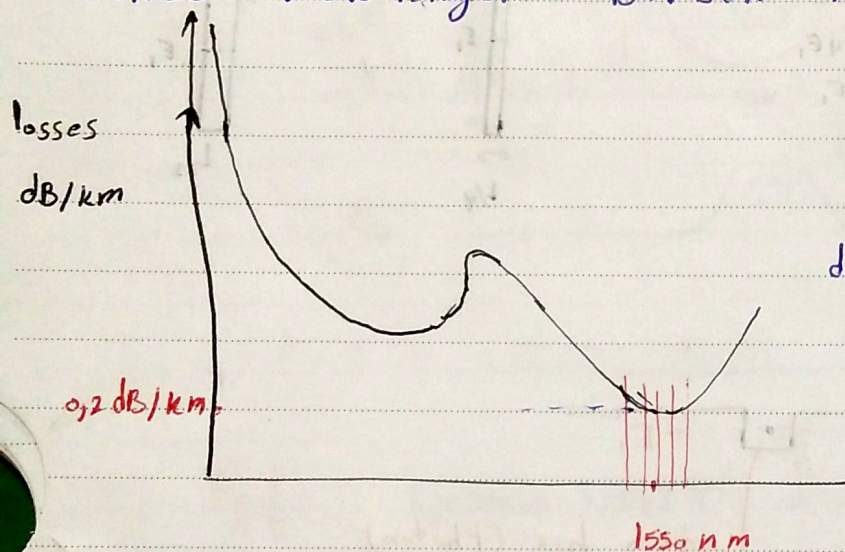
$$E_g = 1,55 - 1,424 = 0,126$$

$$E_{c1} = (0,67 \times 0,126) + 1,55$$

$$E_{v1} = 1,424 - 0,33 \times 0,126$$

Objective: Quantum well (Optical Communication)

Dense Wave length Division multiplexing



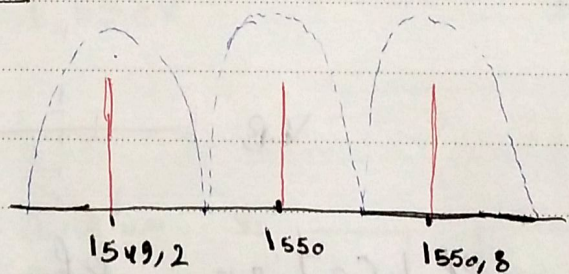
$$f = \frac{c}{\lambda} \Rightarrow f = \frac{c}{\lambda}$$

$$\frac{df}{d\lambda} = -\frac{c}{\lambda^2}$$

$$df = -\frac{c}{\lambda^2} \cdot d\lambda$$

100 GHz

1550 nm



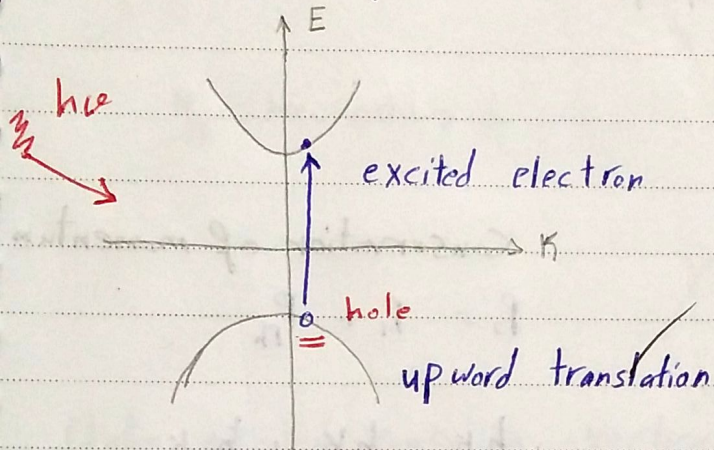
$$0,8 \text{ nm} \rightarrow 100 \text{ GHz}$$

we get it by using
Quantum well structure
because of low value

Lec (12)

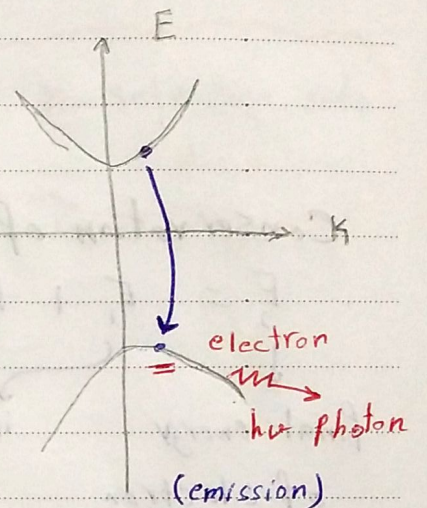
interaction of light with matter

I) - Absorption



app: optical detector (absorption)

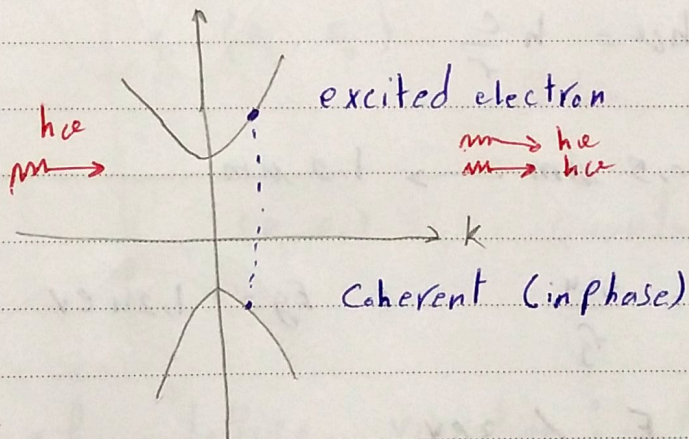
II) Spontaneous Emission



(emission)

app: LED

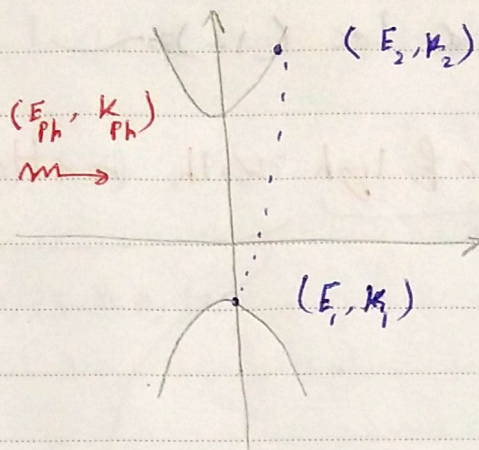
III) Stimulated Emission



downward & upward transition

app: laser

Transition :



Conservation of energy

$$E_2 = E_1 + E_{ph}$$

final energy of electron initial energy of electron

Conservation of momentum

$$p_2 = p_1 + p_{ph}$$

$$\hbar k_2 = \hbar k_1 + \hbar k_{ph}$$

$$k_2 = k_1 + k_{ph}$$

$\downarrow$ $\downarrow$ $\downarrow$
 e Cs ph
 material

$$E_{ph} = h\nu = h \frac{c}{\lambda}$$

$$\lambda = 0,5 \mu m \rightarrow 1,5 \mu m$$

$$E_g = \frac{1,24}{\lambda_g} \quad \text{!} \quad E_g = 1,24 \text{ eV}$$

$$1 \text{ eV} < E_g < 3 \text{ eV}$$

energy of Photon
can allow transition
in semiconductor
material

$$k_2 = k_1 + k_{ph}$$

electron

photon wave

wave vector

Vector

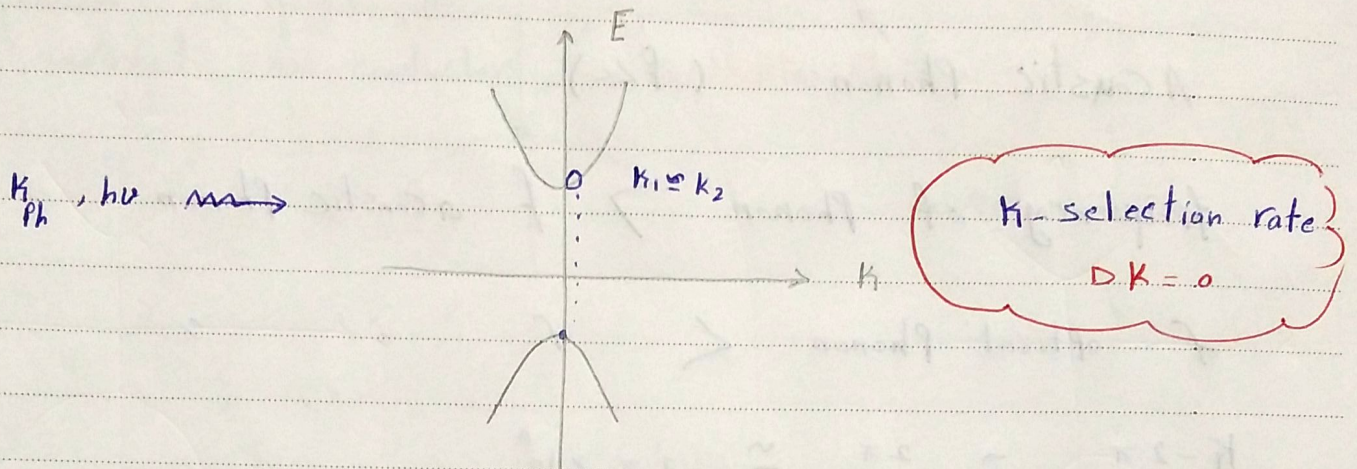
$$\frac{2\pi}{100 \text{ Å}} = \frac{2\pi \times 10^8}{1}$$

$$\frac{2\pi}{1 \times 10^{-6}} = 2\pi \times 10^6 \text{ m}^{-1}$$

μm

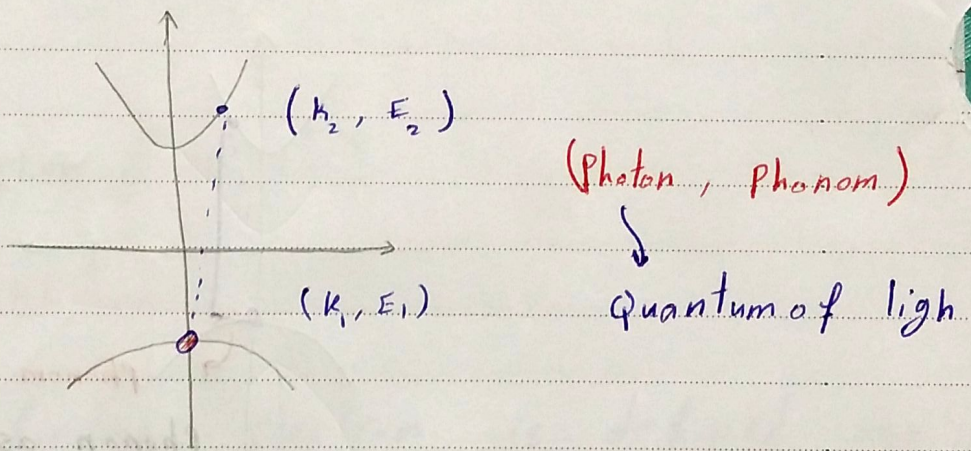
$$(k = \frac{2\pi}{\lambda \text{ Å}})$$

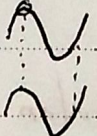
$\Rightarrow k_2 - k_1 \approx 0$ momentum of photon is neglected
w.r.t momentum of electron

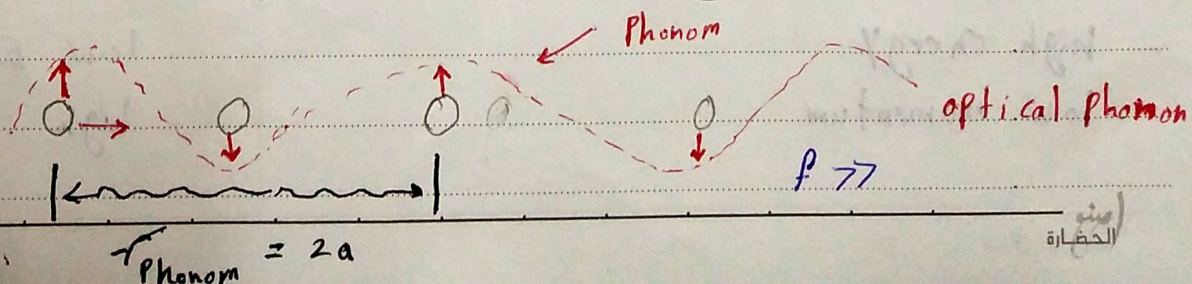
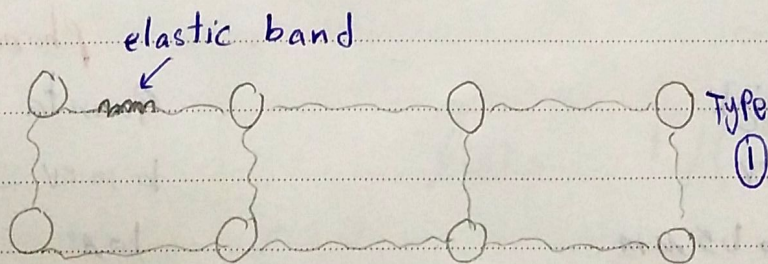


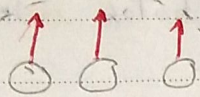
Photon Assisted transition = Vertical transition

oblique transition:



Phonon Quantum of lattice vibration: 





Type ②

Acoustic Phonon ($P \ll$)

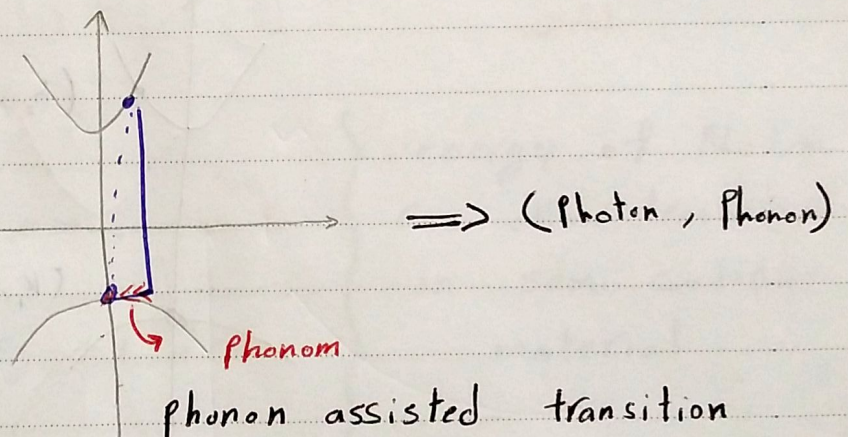
frequency of Phonon $>$ f acoustic Phonon

f optical Phonon $<$ f

$$\frac{K - 2\pi}{\tau_{\text{Phonon}}} \approx \frac{2\pi}{10 \text{ Å}^0} \approx 2\pi \times 10^9$$

Energy of Phonon = small

= 10 meV $\rightarrow$ 70 meV $\approx$ thermal Energy



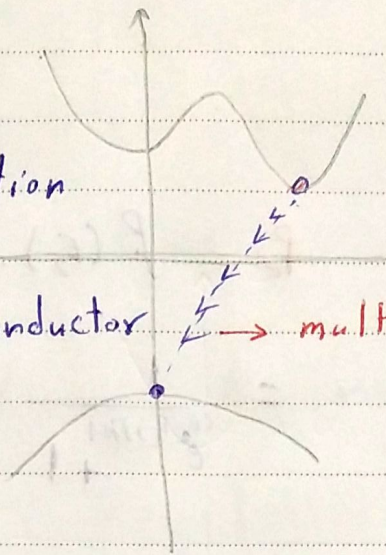
Photon

- Quanta of light
- E: 1 eV $\rightarrow$ 3 eV
- λ : 0.5 μm $\rightarrow$ 1.5 μm
- high energy
- low momentum

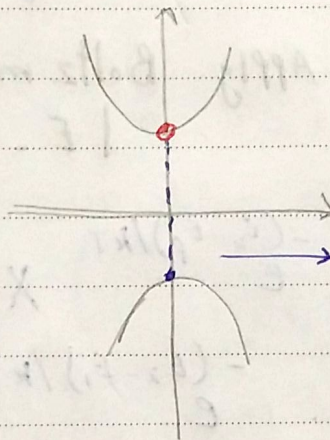
Phonon

- Quanta of lattice vibration
- 10 meV $\rightarrow$ 70 meV
- 10 Å⁰ $\rightarrow$ 100 Å⁰
- low Energy
- high momentum

Probability for radiation of phonon is large for indirect semiconductor



indirect



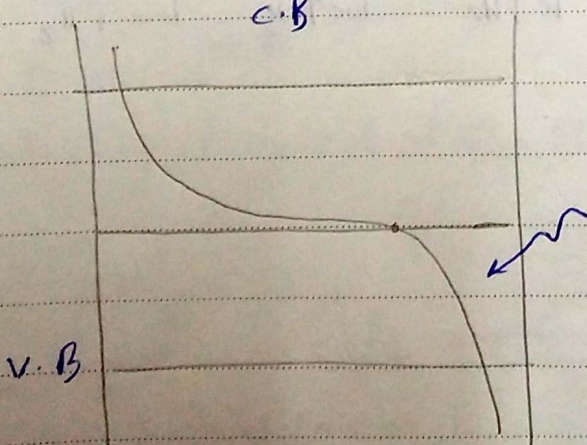
direct

Probability of Absorption & emission:

1). Thermal Equilibrium:

Probability of finding electron is defined one fermi function

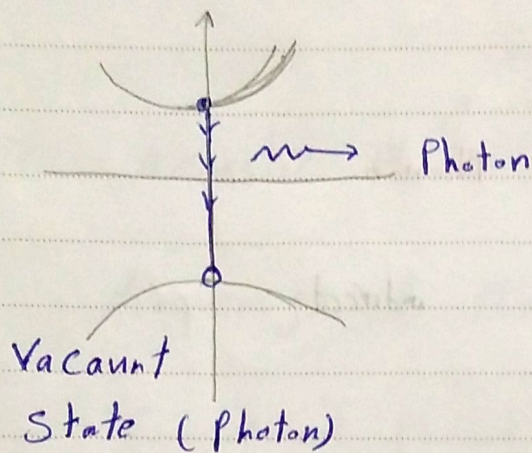
c.B



$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

v.B

$$P_{\text{emission}} = ?$$



$$P_e = P(E_2) = [1 - P(E_1)]$$

$$= \frac{1}{e^{(E_2 - E_f)/kT} + 1} \cdot \left[1 - \frac{1}{e^{(E_1 - E_f)/kT} + 1} \right]$$

$$= \left[\frac{1}{e^{(E_2 - E_f)/kT} + 1} \right] \left[\frac{e^{(E_1 - E_f)/kT}}{e^{(E_1 - E_f)/kT} + 1} \right]$$

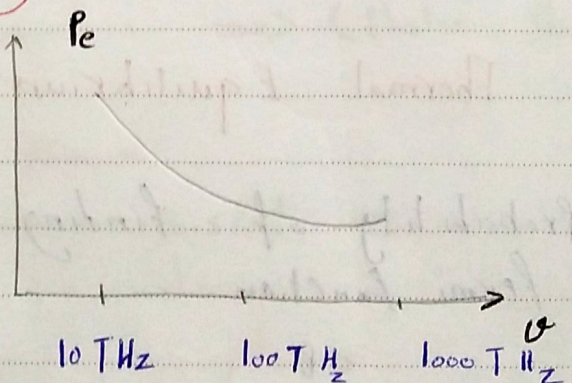
Apply Boltzmann's APPROX
 $|E - E_f| \gg kT$

$$\Rightarrow P_e = \frac{e^{-(E_2 - E_f)/kT}}{e^{-(E_2 - E_f)/kT} + 1} \times \frac{e^{(E_1 - E_f)/kT}}{e^{(E_1 - E_f)/kT} + 1}$$

$$= \frac{e^{-(E_2 - E_1)/kT}}{e^{-(E_2 - E_1)/kT} + 1}$$

$$P_e = \frac{e^{-h\nu/kT}}{e^{-h\nu/kT} + 1} \approx \text{Zero} \quad T = 300 \text{ K} = \text{const}$$

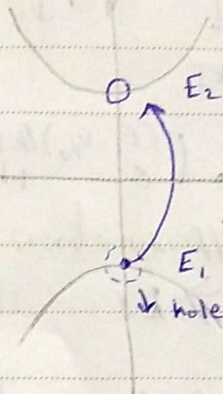
$h\nu = 0.025 \text{ eV}$



$P_{\text{absorption}} =$

electron E_1 exists in V.B

Vacant state E_2 exists in C.B $\rightarrow$



$$P_{ab} = f(E_1) [1 - f(E_2)]$$

$$= \frac{1}{e^{(E_1 - E_F)/kT} + 1} \left(1 - \frac{1}{e^{(E_2 - E_F)/kT} + 1} \right)$$

$$= \left(\frac{1}{e^{(E_1 - E_F)/kT} + 1} \right) \left(\frac{e^{(E_2 - E_F)/kT}}{e^{(E_2 - E_F)/kT} + 1} \right)$$

$$|E - E_F| \gg kT$$

$$P_{ab} \approx 1$$

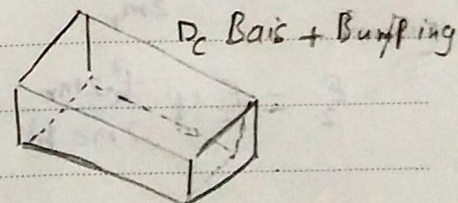
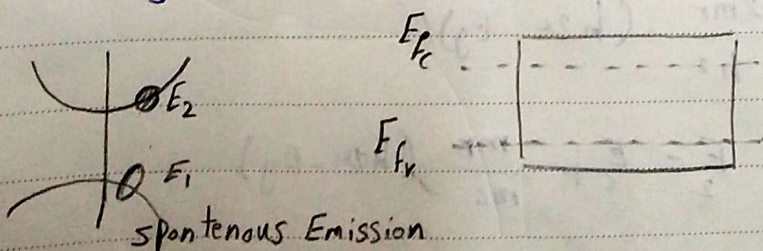
$$P_{abs} > P_e$$

Lec (13)

At thermal equilibrium

$$P_e = e^{-h\nu/kT}$$

The probability of emission at Quasi equilibrium



$$P_e = P_c(E_2) \left[1 - \frac{P_v(E_1)}{P_c(E_2)} \right] = \frac{1}{e^{(E_2 - E_{Fc})/kT} + 1} \times \left[1 - \frac{1}{e^{(E_1 - E_{Fv})/kT} + 1} \right]$$

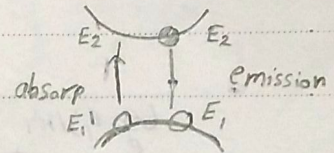
$$= \frac{1 \cdot e^{(E_1 - E_{Fv})/kT}}{(e^{(E_2 - E_{Fc})/kT} + 1) \cdot (e^{(E_1 - E_{Fv})/kT} + 1)}$$

Apply Boltzmann Approximation:

$$(E_2 - E_{Fc}) \gg kT \quad \text{and} \quad |(E_1 - E_{Fv})| \gg kT$$

$$P_e = \frac{e^{-(E_2 - E_{Fc})/kT}}{e} \cdot \frac{e^{(E_{Fc} - E_{Fv})/kT}}{e} = \frac{e^{-(E_2 - E_1)/kT}}{e} \cdot \frac{e^{(E_{Fc} - E_{Fv})/kT}}{e}$$

$$P_e = \frac{e^{-h\nu/kT}}{e} \cdot \frac{e^{(E_{Fc} - E_{Fv})/kT}}{e}$$



Rate of Spontaneous Emission

$$R_{sp} = \frac{1}{\tau_r} \cdot P(\nu) \cdot P_e = \text{No. of emitted photon Per unit time Per unit Volume Per unit energy}$$

\$P(\nu)\$ = optical gain density \$\times \tau_r\$ time of recombination

\$\downarrow\$ no. of (emitted or absorbed photon) Per unit Volume Per unit energy

\$P(\nu)\$

$$E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} \quad (1)$$

$$E_1 = E_v + \frac{\hbar^2 k^2}{2m_v} \quad (2)$$

$$(2) - (1) \Rightarrow$$

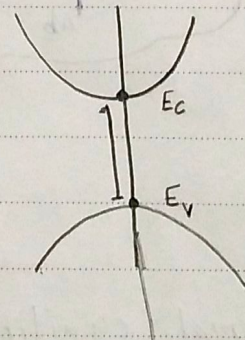
$$h\nu = E_g + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_c} + \frac{1}{m_v} \right) + \frac{1}{m_r}$$

$$h\nu = E_g + \frac{\hbar^2 k^2}{2m_r}$$

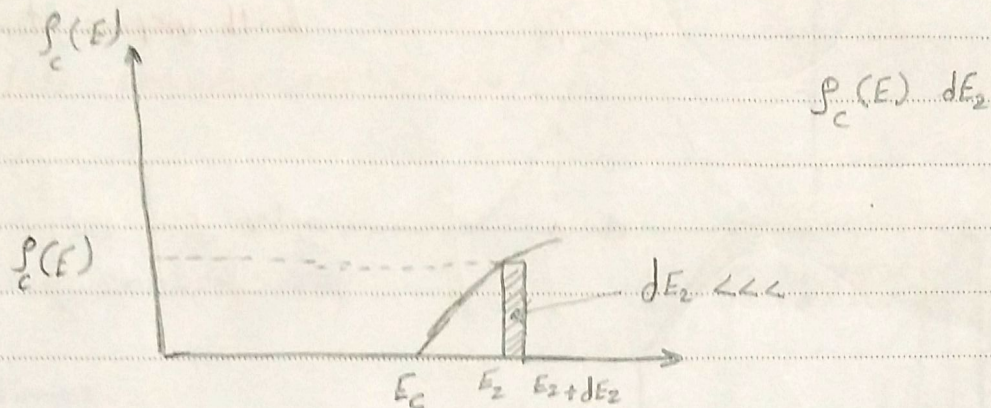
$$k^2 = \frac{2m_r}{\hbar^2} (h\nu - E_g)$$

$$E_2 = E_c + \frac{\hbar^2 \cdot 2m_r}{2m_c \hbar^2} (h\nu - E_g)$$

$$E_2 = E_c + \frac{m_r}{m_c} (h\nu - E_g)$$



No of allowed state per unit volume ($E_2 \rightarrow E_2 + dE_2$)



$$P(E_2) dE_2 = P(u) \cdot du$$

$$P(u) = P_c(E_2) \cdot \frac{dE_2}{du}$$

$$P(u) = \frac{1}{2\pi^2} \left(\frac{2mc}{h^2} \right)^{3/2} (E_2 - E_c)^{1/2} \cdot \frac{m_r h}{m_c}$$

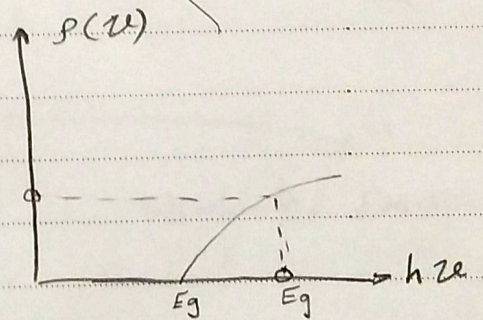
$$(E_2 - E_c) = \frac{m_r}{m_c} (h\nu - E_g)$$

$$P(u) = \frac{1}{\pi h^2} (2m_r)^{3/2} \cdot (h\nu - E_g)^{1/2}$$

$$R_{sp} = \frac{1}{\tau \pi h^2} (2m_r)^{3/2} \cdot (h\nu - E_g)^{1/2} e^{-h\nu/kT}$$

$$= \frac{1}{\tau \pi h^2} (2m_r)^{3/2} e^{-E_g/kT} \left[(h\nu - E_g)^{1/2} \cdot e^{-(h\nu - E_g)/kT} \right]$$

$$P = \frac{1}{kT}$$

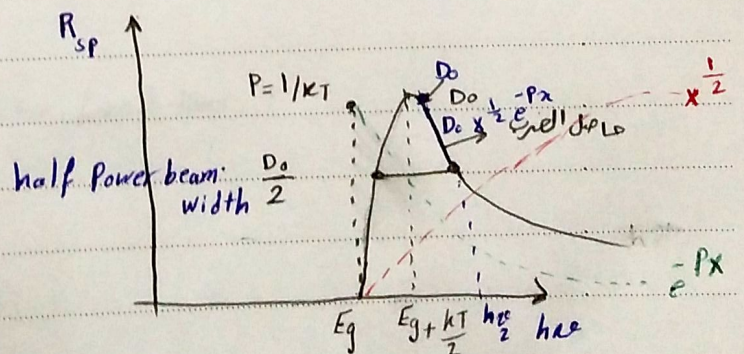


$$R_{sp} = D_0 \cdot x^{1/2} e^{-Px}$$

$$\frac{dR_{sp}}{d h\nu} = 0 \Rightarrow [h\nu] \text{ peak value}$$

$$\Delta f = 1,45 \text{ } K T f_p^2$$

$$\Delta \nu = 1,45 \text{ } K T c$$



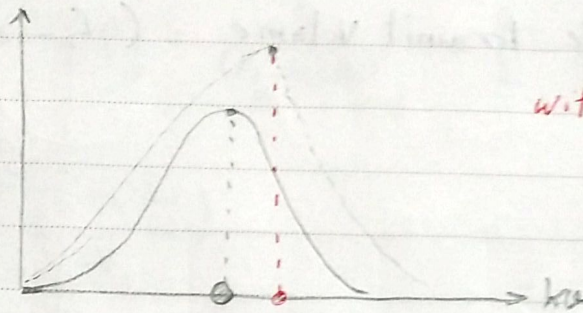
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میراث

Comp
msq

given reason
draw

/ /



with increas temperature

Lec (14)

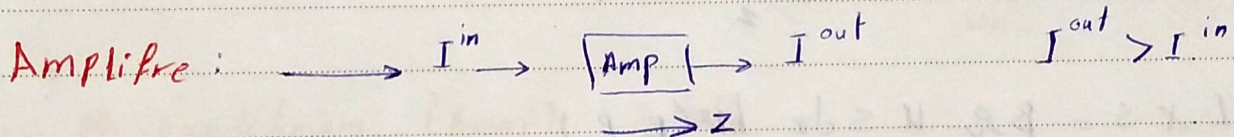
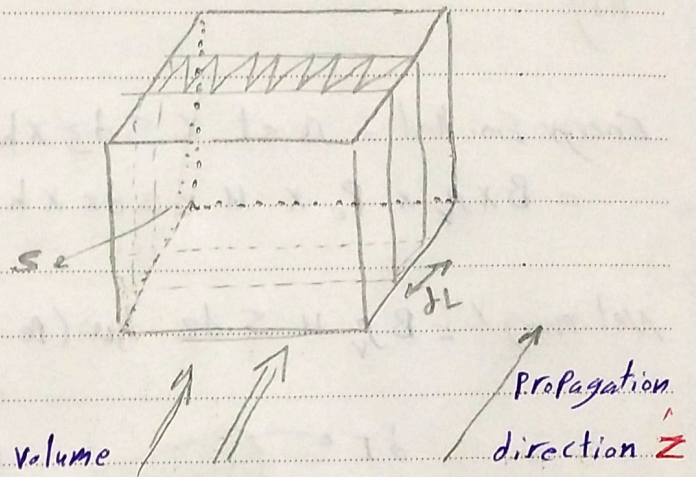
Stimulate Emission Amplification:

S Cross section Area

$$\text{Power} = \frac{\text{Energy}}{\text{unit time}}$$

$$I_{\text{radiance}} = \frac{\text{Power}}{\text{unit Area}} = I$$

$$\text{Energy density } u = \text{energy/unit volume}$$



Amplify

Stimulation

Stimulated Emission

① Absorption

$$\text{rate of Absorption} = \kappa_{ab}$$

$$\kappa_{ab} = B \times \rho_r \times P_{abs} \times u$$

$$\kappa_{ab} = \text{no of absorbed photon/unit volume/unit time}$$

$$B = \frac{u^3}{8\pi h \epsilon_r u^3}$$

u = speed of wave inside material

u = frequency of incident wave

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$$\text{No of absorbed photon in } S \cdot dz = B \rho_v \rho_{abs} u \cdot S \cdot dz$$

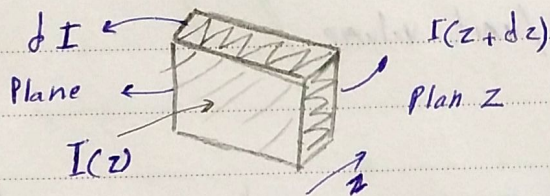
$$\text{energy absorbed in } S \cdot dz = B \rho_v \rho_{abs} \cdot u \cdot S \cdot dz \cdot h\nu$$

Similarly

$$\begin{aligned} \text{Energy Emitted} &= \rho_{st} \times S \cdot dz \times h\nu \\ &= B \rho_v \times \rho_e \times u \times S \cdot dz \times h\nu \end{aligned}$$

$$\text{Net energy} = B \rho_v u S \cdot dz h\nu (\rho_e - \rho_{ab})$$

/unit time



$$dI \times S = B \rho_v u S \cdot dz h\nu (\rho_e - \rho_{ab})$$

$$\frac{dI}{dz} = B \rho_v u h\nu (\rho_e - \rho_{ab})$$

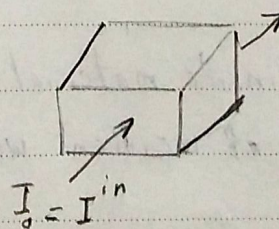
$$u = \frac{I}{\rho_e}$$

$$\frac{dI(z)}{dz} = \underbrace{\frac{B \rho_v h\nu}{c}}_{\text{speed}} (\rho_e - \rho_{ab}) I(z)$$

γ

$$\frac{dI(z)}{dz} = \gamma I(z) \Rightarrow I(z) = I_0 e^{\gamma z}$$

$$c = \frac{c}{n}$$



$$I^o = I_0 e^{-\gamma \delta z}$$

δ

$\delta = +ve$

Amplify

$\delta = \text{Gain}$

$\delta = -ve$

Attenuation

$\delta = \text{loss}$

$$\delta = \frac{B P_v h\nu}{I_0} (P_e - P_{abs})$$

$$P_v = \frac{1}{\pi \hbar^2} (2m_v)^{\frac{3}{2}} (h\nu - E_g)^{\frac{1}{2}}$$

$h\nu > E_g$

$$B = \frac{ce^3}{8\pi h\nu^3}$$

Equasi Equilibrium (bump) $P_e > P_{abs}$



two fermi level E_{fc} , E_{fv}

two function

$$P_e = f_c(E_2) \times [1 - f_v(E_1)] = \frac{1}{\frac{(E_2 - E_{fc})}{kT} + 1} \times \frac{e^{\frac{(E_1 - E_{fv})}{kT}}}{\frac{(E_2 - E_{fv})}{kT} + 1}$$

$$P_{abs} = f_v(E_1) \times [1 - f_c(E_2)] = \frac{1}{\frac{(E_1 - E_{fv})}{kT} + 1} \times \frac{e^{\frac{(E_2 - E_{fc})}{kT}}}{\frac{(E_2 - E_{fc})}{kT} + 1}$$

$P_e > P_{ab}$

$(E_1 - E_{fv}) > E_2 - E_{fc}$

$(E_2 - E_1) < (E_{fc} - E_{fv})$

$E_g < h\nu < (E_{fc} - E_{fv})$

$\frac{E_g}{h} < \nu < \left(\frac{E_{fc} - E_{fv}}{h}\right)$

$$P_e - P_{ab} = f_c(E_2) (1 - f_v(E_1)) - f_v(E_1) (1 - f_c(E_2))$$

$$P_e - P_{ab} = f_c(E_2) - f_v(E_1)$$

$$\delta = \frac{\omega^3}{8\pi\hbar\omega^3} \times \frac{1}{\pi\hbar^2} (2mr)^{\frac{3}{2}} \cdot \frac{\hbar\omega}{v} (\hbar\omega - E_g)^{\frac{1}{2}} (f_c(E_2) - f_v(E_1))$$

$$= \frac{\omega^2 (2mr)^{\frac{3}{2}}}{2\hbar^2\omega^2} (\hbar\omega - E_g)^{\frac{1}{2}} (f_c(E_2) - f_v(E_1))$$

$$\delta = \frac{(c/\lambda)^2}{2\pi r} \cdot \frac{(2mr)^{\frac{3}{2}}}{\hbar\omega^2} (\hbar\omega - E_g)^{\frac{1}{2}} (f_c(E_2) - f_v(E_1))$$

EX: at $T = 0K$

a) - Thermal Equilibrium :

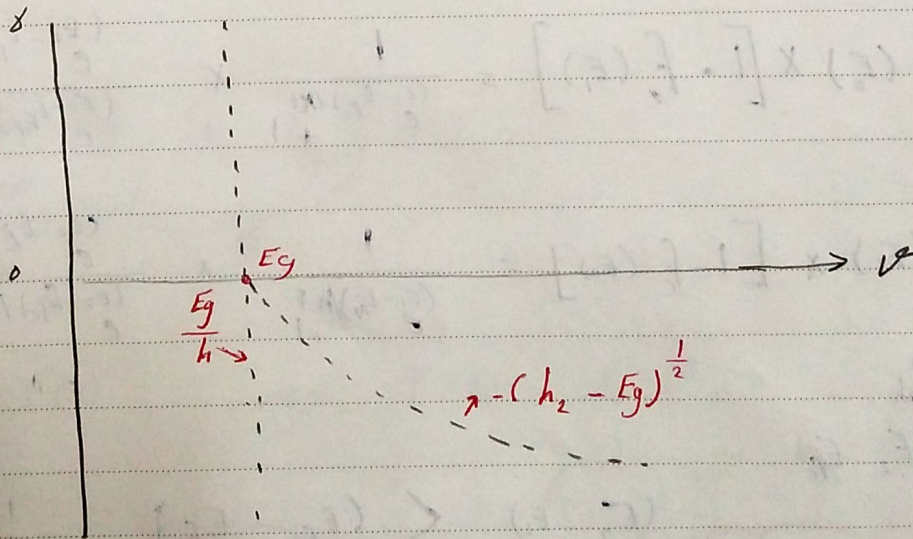
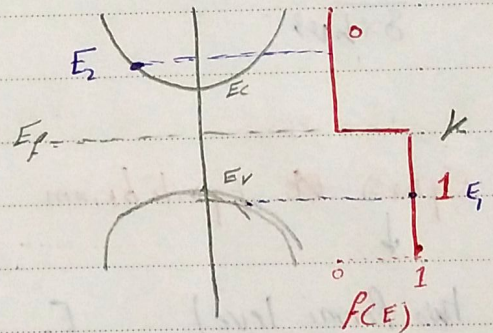
$$f(E) = \frac{1}{\frac{(E - E_f)/kT}{B} + 1}$$

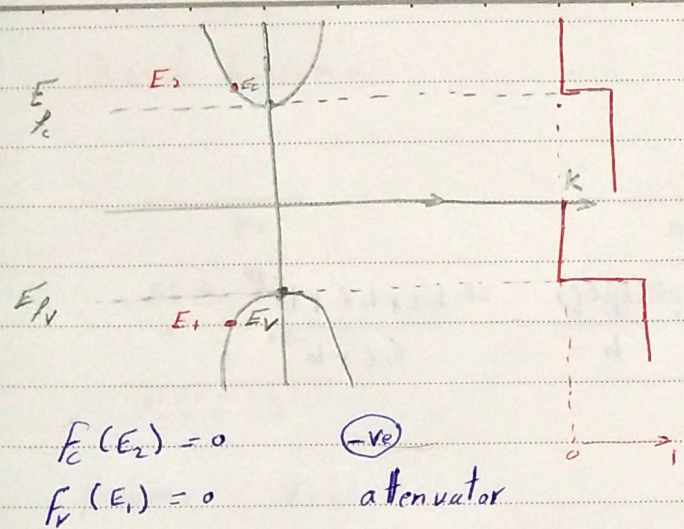
$$f(E_2) = \text{Zero}$$

$$f(E_1) = 1$$

$$f(E_2) - f(E_1) = -1$$

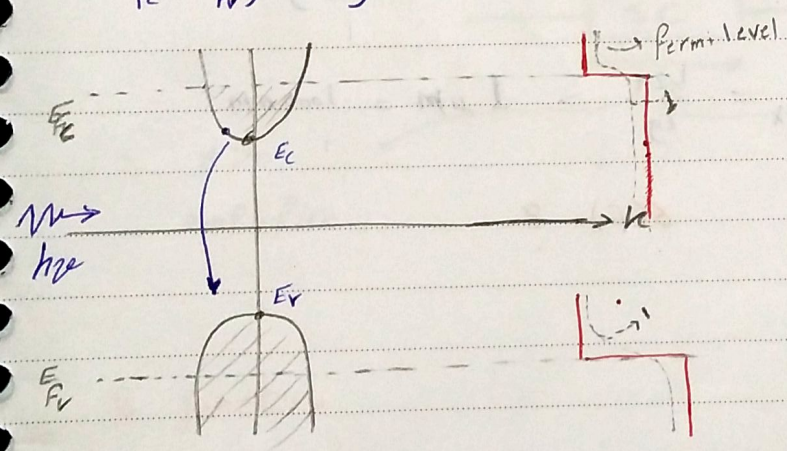
$$\delta = -ve$$





Quasi equilibrium

$$(F_c - F_v) > E_g$$

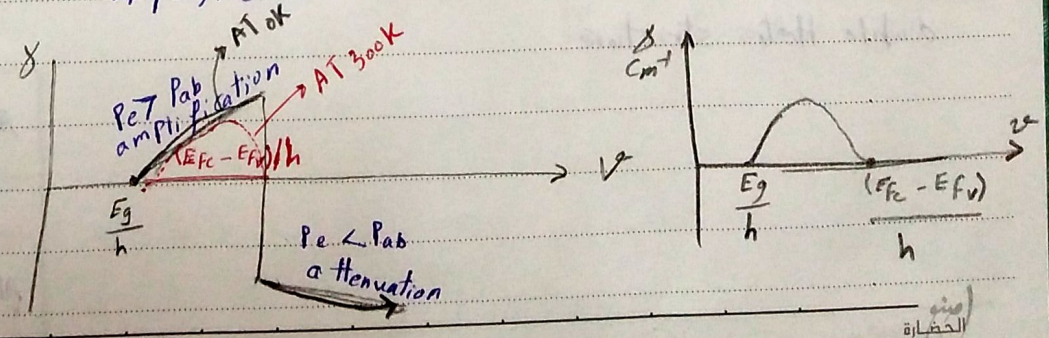


$$F_c(E_2) = 1$$

$$F_v(E_1) = 0$$

$$F_c(E_2) - F_v(E_1) = +1$$

$\delta = +ve$
 amplification
 amplification in



EX:

$$E_g = 1 \text{ eV}$$

$$\frac{E_c - E_{fv}}{E_g} \approx 1/2$$

$$\star \frac{E_g}{h} = \frac{1 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 0.25 \times 10^{15} \text{ Hz}, \quad \frac{(E_c - E_{fv})}{h} = \frac{1/2 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$$

⇒ Range of operation (BW)

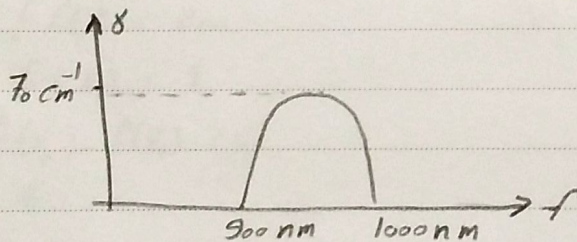
$$250 \text{ THz} \longrightarrow$$

cel Lec 15 130

EX: give semiconductor Amplifier made of InGaP with $E_g = 1.24 \text{ eV}$
peak coefficient gain = 70 cm^{-1} & $\text{BW} = 100 \text{ nm}$

Draw the Variation of the amplification coeff with wavelength

SOL: $E_g = 1.24 \text{ eV} \Rightarrow \lambda_{\text{max}} = \frac{1.24}{E_g} = 1 \mu\text{m} = 1000 \text{ nm}$



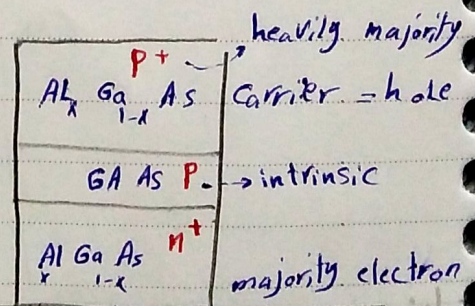
$\delta(f) ?$

$$\delta > 0 \quad \text{and} \quad P_e > P_{\text{abs}}$$

$$(E_c - E_{fv}) > E_g$$

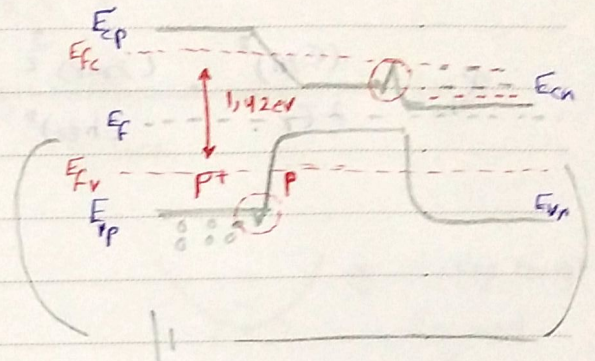
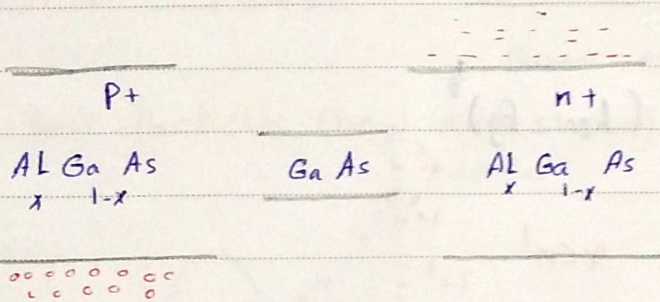
Man Structure:

double Hetro structure



Band diagram

steady state

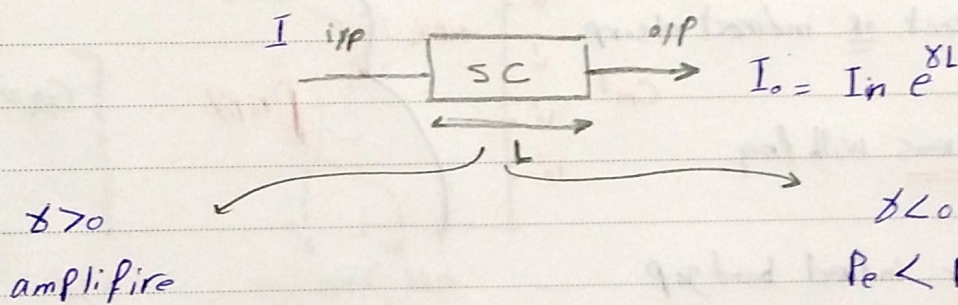


Assume $x=0.1$

$$E_g = 1.5 \text{ eV}$$

$$E_g = 1.42 \text{ eV}$$

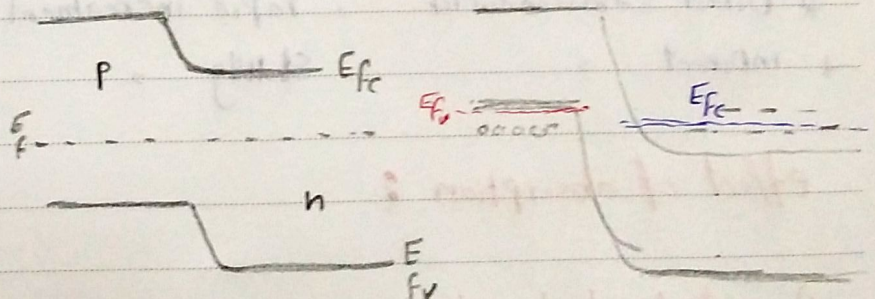
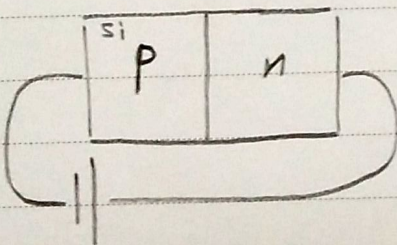
Attenuation :



$$P_e < P_{ab}$$

$$P_e - P_{abs} = (P_e(E_2) - P_v(E_1)) < 0$$

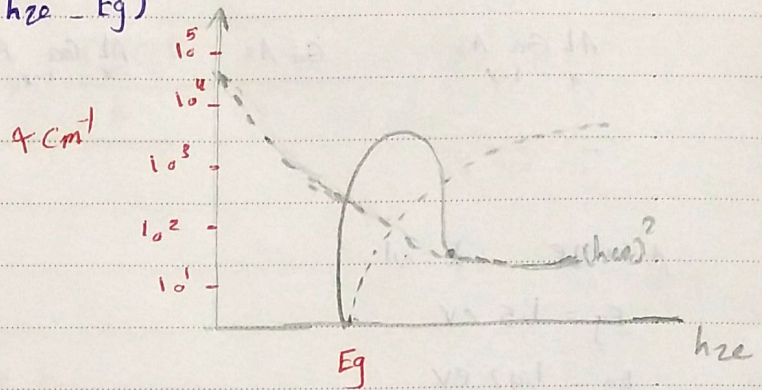
Pn junction



Absorption Spectrum :

$$x = -\alpha_{abs}$$

$$\alpha_{abs} = \frac{(C/n)^2}{2\tau r} \cdot \frac{(2\pi r)^{\frac{3}{2}}}{(h\nu)^2} \cdot (h\nu - E_g)^{\frac{1}{2}}$$

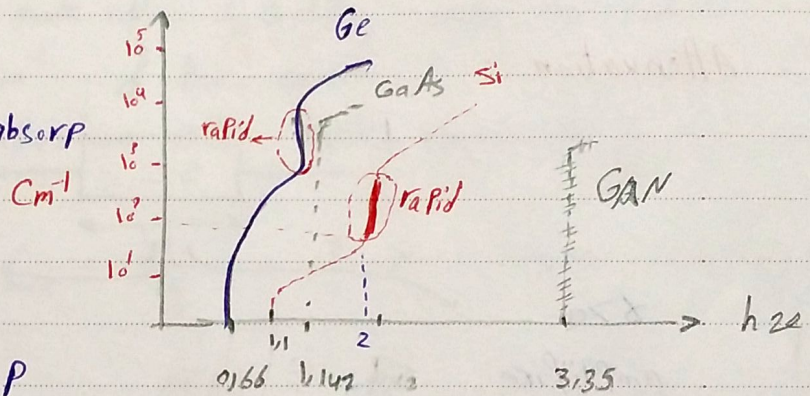


typical Absorption Spectrum:

* Any material Direct OR indirect absorp

* Absorption increases with freq

* Curves Consider indirect band gap



$$\alpha(\text{cm}^{-1}) = (K_0 + K_T) (h\nu - E_g)^2$$

* Direct semiconductor , rapid increment

* indirect = , slowly =

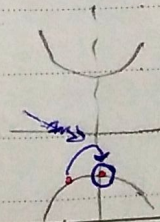
effect of absorption ?

1) interband transition

electron excited from v.B to c.B

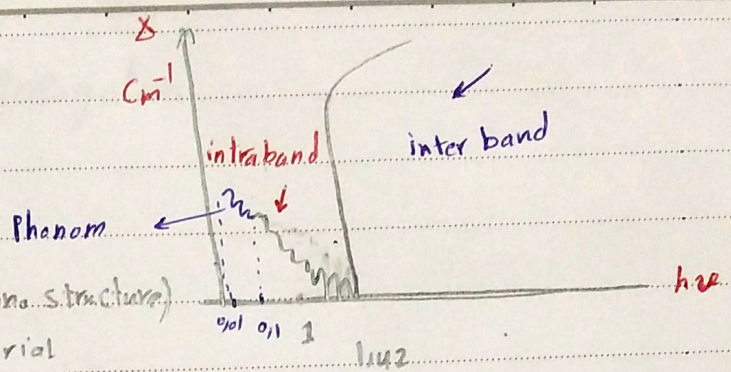
2) intraband transition:

electron excited in the same band (from position to another)



3) - Phonon

0.01 eV $\rightarrow$ 0.1 eV



4) Excitonic Particles (report in nano structure material)

